Student Name: ______________________

HUNTERS HILL HIGH SCHOOL
MATHEMATICS
HSC TRIAL 2016

Hunters Hill High School

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A Reference Sheet is provided
- In Questions 11–16, show relevant mathematical reasoning and/or calculations

Total marks - 100

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt Questions 11-16
- Allow about 2 hour 45 minutes for this section

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 Round $\sqrt{4^2 + 5^2 - 2 \times 4 \times 5 \times \cos\frac{\pi}{6}}$ to 3 significant figures

(A) 1.00

(B) 1.001

(C) 2.52

(D) 2.521

2 The domain of $\frac{1}{\sqrt{9-x^2}}$ is

(A) $-3 < x < 3$

(B) $x > 3 \; or \; x < -3$

(C) $-3 \leq x \leq 3$

(D) $x \geq 3 \; or \; x \leq -3$

3 $\log_4 12$ is numerically equivalent to

(A) $\frac{\log_{12} 4}{\log_{12} 3}$

(B) $\frac{\log_4 12}{\log_4 3}$

(C) $\frac{\log_e 12}{\log_e 4}$

(D) $\frac{\log_e 4}{\log_e 12}$

4 Hens in a barnyard lay eggs such that 55% are white and 45% are brown. If two eggs are selected at random, what is the probability they are both white?

(A) 0.2025

(B) 0.2475

(C) 0.3025

(D) 0.5555

5 The limiting sum of the series $x + x^2 + \cdots$. is 15. If $|x| < 1$, the value of x is

(A) $-\frac{16}{15}$

(B) $-\frac{15}{16}$

(C) $\frac{16}{15}$

(D) $\frac{15}{16}$

6 The solution to $|2x + 5| < 3$ is

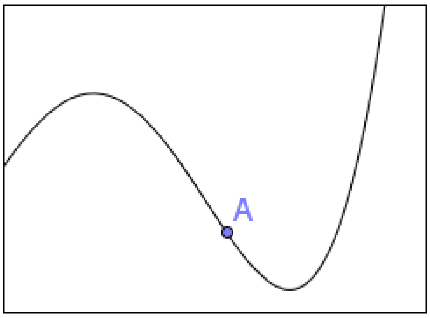
(A) $-4 < x < -1$

(B) $x < -4, \ x > -1$

(C) $-1 < x < 4$

(D) $x < --1, \ x > 4$

7 The function f(x) is shown below. Which of the following is true at the point A?



(A) $f'(x) > 0, f''(x) > 0$

(B) $f'(x) > 0, f''(x) < 0$

(C) $f'(x) < 0, f''(x) > 0$

(D) $f'(x) < 0, f''(x) < 0$

8 The Amplitude and Period for $y = 3\cos 2x$ are, respectively,

(A) $2,\ \frac{2\pi}{3}$

(B) $3,\ \pi$

(C) $\pi,\ 3$

(D) $\frac{2\pi}{3},\ 2$

9 A particle is moving along the *x*-axis. The displacement of the particle after *t* seconds is given by $x = t^2 - 3t$ metres. Which statement describes the motion after 1 s?

(A) The particle is moving to the left with decreasing speed.

(B) The particle is moving to the right with decreasing speed.

(C) The particle is moving to the left with increasing speed.

(D) The particle is moving to the right with increasing speed

10 Let α and β be the roots of the equation $2x^2 - 5x - 9 = 0$. The value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is

(A) $-\frac{9}{2}$

(B) $-\frac{9}{5}$

(C) $-\frac{5}{9}$

(D) $\frac{5}{2}$

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) Express $\frac{6}{2-\sqrt{7}}$ in the form $a + b\sqrt{7}$ **2**

(b) Evaluate **2**

$$\lim_{x\to\infty} \frac{3x^2 - 4x + 5}{x^2}$$

(c) Matt bought some tickets in a raffle that offered two prizes. 100 tickets were sold in total. Two tickets were drawn without replacement to determine the prize-winners. The probability that Matt won both prizes was $\frac{2}{275}$.
Find the number of tickets that Matt bought. **2**

(d) Differentiate with respect to *x*

(i) $x\ lnx$ **1**

(ii) $\frac{2e^x}{x^2+5}$ **2**

(iii) cos^2x **2**

(e) Evaluate **2**

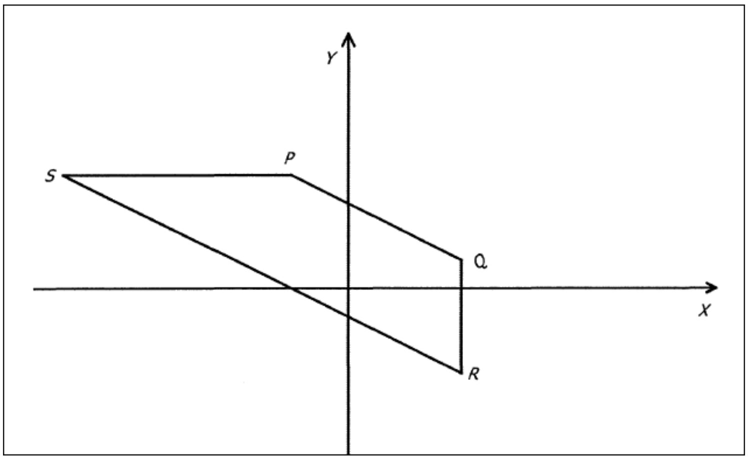
$$\int_0^{\frac{\pi}{4}} \sin 2x\, dx$$

(f) Find **2**

$$\int \frac{x}{x^2+3}\, dx$$

Question 12 (15 marks)

(a) In the quadrilateral *PQRS* the coordinates of the points *P* and Q are (-2,4) and (4,1) respectively. The equation of line *SR* is $x + 2y + 2 = 0$.



(i) Find the gradients of *PQ* and *RS*. Hence, explain why the quadrilateral *PQRS* is a trapezium. **2**

(ii) Find the length of *PQ* in exact form. **2**

(iii) The line *QR* is parallel to the y axis, find the coordinates of point *R*. **1**

(iv) Find the perpendicular distance from *P* to the line *RS*. **2**

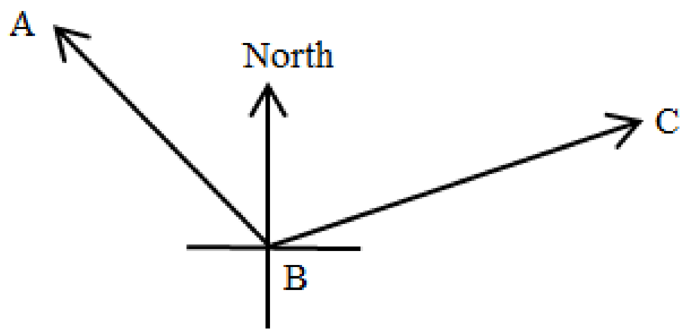
(v) If the length of *RS* is $\sqrt{85}$ units find the area of trapezium *PQRS*. **2**

(b) The table shows the values of a function $f(x)$ for five values of *x*. **2**

x	1	1.5	2	2.5	3
f(x)	4	1.5	2	2.5	8

Use Simpson's Rule with these five values to estimate $\int_1^3 f(x)dx$

(c) Two bushwalkers, Aaron and Charlotte leave Point B at the same time. Aaron walks on a bearing of 310^{o} at a speed of 1.8 km/h and Charlotte walks on a bearing of 070^{o} at a speed of 2.4 km/h. Copy the diagram below onto your answer page and mark the information on the diagram.



(i) How far apart are Aaron and Charlotte after two hours? Answer to 1 decimal place. **2**

(ii) What is the bearing of Charlotte from Aaron after 2 hours? **2**

Question 13 (15 marks)

(a) A sports club gained 50 members on its first day of operation. As the club's popularity grew, clientele grew steadily by an extra14 members every subsequent day (ie 64 new members the second day, 78 new members on the third day and so on).

(i) How many members did the club have join on its 28th day of operation?

(ii) After how many days of operation did the club have over 800 members join on one single day of operation? **2**

(iii) On which day did the 10 000th member join the club? **2**

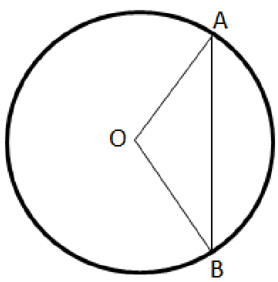
(b) Consider the parabola $2y = x^2 - 4x$.

(i) Rewrite it in the form $4a(y-k) = (x-h)^2$ **2**

(ii) Give the coordinates of the focus. **1**

(iii) Give the equation of the directrix. **1**

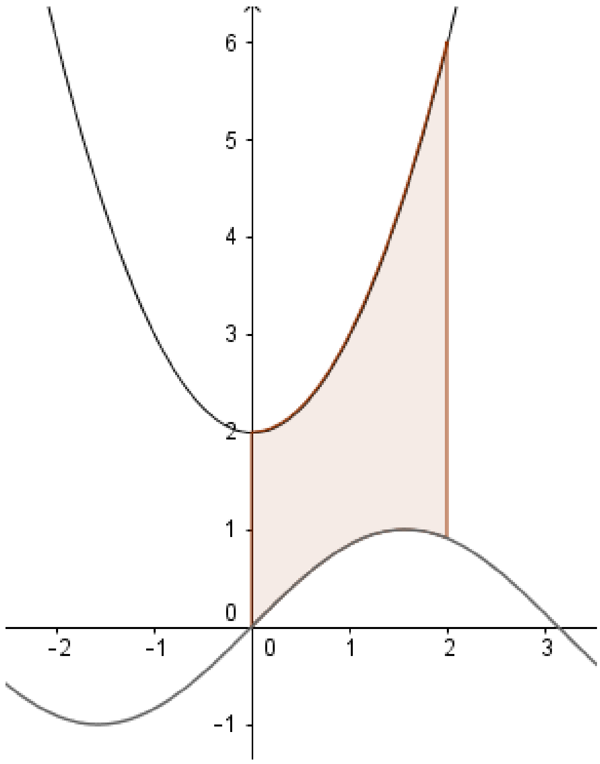
(c) A circle has centre O and radius 8cm. The length of arc AB is 6π.



(i) Find the size of $\angle AOB$. Answer in radians. **2**

(ii) Find the area of the minor segment cut off by the chord AB. Give your answer to 1 decimal place. **2**

(d) Find the area enclosed by the curves $y = x^2 + 2$, $y = \sin x$ and the ordinates $x = 0$ and $x = 2$. Answer to 2 decimal places. **3**



Question 14 (15 marks)

(a) For the curve $y = 2x^3 + 3x^2 - 12x - 9$

(i) Find any stationary points and determine their nature. **3**

(ii) Find the point of inflexion **2**

(iii) Sketch the curve in the domain $-3 \leq x \leq 3$ showing the y-intercept **2**

(iv) Find the minimum value of the curve in the domain $-3 \leq x \leq 3$ **1**

(b) Emily borrows $750 000 to purchase her first home. She takes out a loan over 30 years, to be repaid in equal monthly instalments. The interest rate is 4.8% *per annum*, calculated *monthly*. Let A_n be the amount owing at the end of n months and M be the monthly repayment.

(i) Show that $A_2 = 750\,000(1.004)^2 - M(1 + 1.004)$ **1**

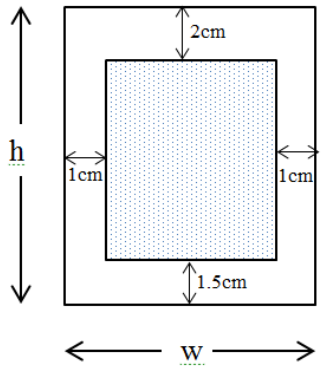
(ii) Show that $A_n = 750\,000(1.004)^n - M(\frac{(1.004)^n - 1}{0.004})$ **2**

(iii) Find the monthly repayment required to repay the loan in 30 years. **2**

(iv) Emily wants pay the loan off in less than 30 years. If she can afford to pay $5 000 per month, how many months will it take her to pay off the home loan? **2**

Question 15 (15 marks)

(a) A printer needs to make a poster that will have a total area of 200 cm^2 and will have 1 cm margins on the sides, a 2 cm margin on the top and a 1.5 cm margin on the bottom.



(i) Show that the area of the shaded region is **2**

$$A = 207 - 3.5w - \frac{400}{w}$$

(ii) Find the dimensions of the paper, h and w (to 2 decimal places), for which the area of the shaded region will be a maximum. **3**

(b) Min needs to take a drug to control a medical condition. It is known that the quantity Q of drug remaining in the body after t hours satisfies an equation of the form $Q = Q_0 e^{-kt}$ where Q_0 and k are constants.

The initial dose she takes is 5 milligrams and after 12 hours the amount remaining in her body is half the initial dose.

(i) Find the values of Q_0 and k. **3**

(ii) How long until only 0.5 milligrams of the drug remains in Min's body? (Answer to the nearest hour) **2**

(c) A particle moves on a horizontal line so that its displacement is given by the equation $x = t^3 - 12t^2 + 36t - 8$ where t is measured in seconds and x in metres.

(i) When does the particle come to rest? **2**

(ii) When does the particle first change direction? **1**

(iii) Find the total distance travelled by the particle in the first 10 seconds. **2**

Question 16 (15 marks)

(a) Prove that **3**

$$\frac{\sin^2 x}{1-\cos x}+\frac{\sin^2 x}{1+\cos x}=2$$

(b) The rate at which Carbon Dioxide will be produced when conducting an experiment is given by $\frac{dV}{dt}=\frac{1}{100}\left(30t-t^2\right)$ where V cm^3 is the volume of gas produced after t minutes. There is initially no Carbon Dioxide present.

(i) At what rate is the gas being produced 15 minutes after the experiment begins? **1**

(ii) How much Carbon Dioxide has been produced during this time? **2**

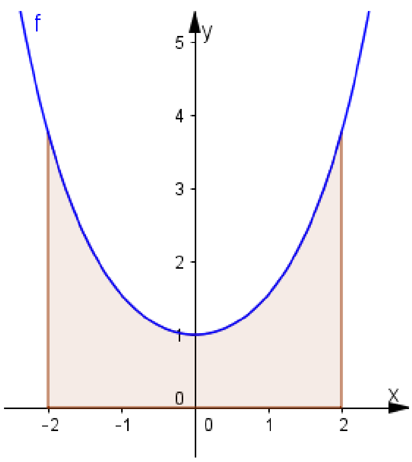
(c) The acceleration of a particle is given by $\ddot{x} = 4\sin 2t$ where x is the displacement in metres and t is the time in seconds. Initially the particle is stationary at $x = 4$.

(i) Show that the velocity of the particle is given by $\dot{x} = -2\cos 2t + 2$ **2**

(ii) Find the time when the particle first comes to rest **2**

(iii) Find the displacement x of the particle in terms of t **2**

(d) Calculate the volume of the solid of revolution when the function $f(x) = \frac{1}{2}(e^x + e^{-x})$ is rotated about the x-axis between the ordinates $x = -2$ and $x = 2$. **3**



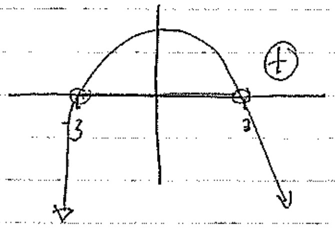
2U MATHS TRIAL 2016

1) $2.5217 = 2.52$ (C)

2) $9 - x^2 > 0$

$(3+x)(3-x) > 0$

$-3 \qquad 3$



$-3 < x < 3$ (A)

3) $\log_4 12 = \dfrac{\log_e 12}{\log_e 4}$ (C)

4) $0.55 \times 0.55 = 0.3025$ (C)

5) $S_\infty = \dfrac{a}{1-r}$ $\quad a = x, \; r = x$

$\dfrac{x}{1-x} = 15$

$x = 15(1-x)$

$16x = 15$

$x = \dfrac{15}{16}$ (D)

6) $|2x+5| < 3$

$+(2x+5) < 3$

$2x < -2$

$x < -1$

$-(2x+5) < 3$

$-2x < 8$

$x > -4$

$-4 < x < -1$ (A)

7) Neg Slope so $f'(x) < 0$

Concave Up so $f''(x) > 0$ (C)

8) $3\cos 2x$ $\quad$ Amp $= 3$

Period $= \pi$ (B)

9) $x = t^2 - 3t$

At $t = 1$ $\quad x = 1^2 - 3$

$= -2$

$\dot{x} = 2t - 3$ $\quad$ Moving to left

$= 2(1) - 3$

$= -1$ at $t = 1$

$\ddot{x} = 2$ $\quad$ Increasing Speed. (C)

10) $2x^2 - 5x - 9 = 0$

$a = 2, \; b = -5 \; c = -9$

$\alpha + \beta = \dfrac{-b}{a}$ $\qquad \alpha\beta = \dfrac{c}{a}$

$= \dfrac{5}{2}$ $\qquad = \dfrac{-9}{2}$

$\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\beta + \alpha}{\alpha\beta}$

$= \dfrac{5/2}{-9/2}$

$= \dfrac{-5}{9}$ (C)

(3) 4

Question 11

a) $\frac{6}{2-\sqrt{7}} \times \frac{2+\sqrt{7}}{2+\sqrt{7}} = \frac{6(2+\sqrt{7})}{4-7}$

$= \frac{12+6\sqrt{7}}{-3}$

$= -4 - 2\sqrt{7}$ (2)

b) $\lim_{x \to \infty} \frac{3x^2 - 4x + 5}{x^2}$

$= \lim_{x \to \infty} 3 - \frac{4}{x} + \frac{5}{x^2}$

$= 3$ (2)

c) Let no. tickets Matt bought be x

$\frac{x}{100} \times \frac{(x-1)}{99} = \frac{2}{275}$

$\frac{x^2 - x}{9900} = \frac{2}{275}$

$x^2 - x = 72$

$x^2 - x - 72 = 0$ ∴ Matt bought 9 tickets

$(x-9)(x+8) = 0$

$x = 9$ $x = -8$ No Sol'n (2)

d) i) $\frac{d}{dx} x \ln x$

$u = x$ $v = \ln x$

$u' = 1$ $v' = \frac{1}{x}$

$\frac{d}{dx}(x \ln x) = u'v + v'u$

$= 1 \cdot \ln x + \frac{1}{x} \cdot x$

$= \ln x + 1$ (1)

ii) $\frac{d}{dx}\left(\frac{2e^x}{x^2+5}\right)$

$u = 2e^x$ $v = x^2+5$

$u' = 2e^x$ $v' = 2x$

$\frac{d}{dx}\left(\frac{2e^x}{x^2+5}\right) = \frac{u'v - v'u}{v^2}$

$= \frac{2e^x(x^2+5) - 2x \cdot 2e^x}{(x^2+5)^2}$

$= \frac{2e^x[x^2+5-2x]}{(x^2+5)^2}$

$= \frac{2e^x(x^2-2x+5)}{(x^2+5)^2}$ (2)

iii) $\frac{d}{dx} \cos^2 x = -2 \sin x \cdot \cos x$ (2)

e) $\int_0^{\pi/4} \sin 2x \, dx$

$= \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/4}$

$= -\frac{1}{2}\left[\cos\frac{2\pi}{4} - \cos\phi\right]$

$= -\frac{1}{2}\left[\cos\frac{\pi}{2} - \cos\phi\right]$

$= -\frac{1}{2}\left[0 - 1\right]$

$= \frac{1}{2}$ (2)

f) $\int \frac{x}{x^2+3} \cdot dx$

$= \frac{1}{2}\int \frac{2x}{x^2+3} \cdot dx$

$= \frac{1}{2}\ln(x^2+3) + c$ (2)

Question 12

a) i) $P\underset{x_1\ y_1}{(-2, 4)}$ $Q\underset{x_2\ y_2}{(4, 1)}$

$m_{PQ} = \frac{1-4}{4--2}$

$= \frac{-3}{6}$

$= -\frac{1}{2}$

$m_{RS} = -\frac{1}{2}$

since $x + 2y + 2 = 0$

$2y = -x - 2$

$y = -\frac{1}{2}x - 1$

PQRS is a trapezium as PQ || SR (2 opposite sides are parallel) (2)

ii) $d_{PQ} = \sqrt{(4--2)^2 + (1-4)^2}$

$= \sqrt{6^2 + (-3)^2}$

$= \sqrt{36+9}$

$= \sqrt{45}$ (2)

iii) Q is (4, 1) $\therefore$ R is $\underset{x\ y}{(4, y)}$

Eqn SR $x + 2y + 2 = 0$ R satisfies this eqn

$\therefore 4 + 2y + 2 = 0$

$2y = -6$

$y = -3$

$\therefore R(4, -3)$

(iv) $d_{perp} = \dfrac{|ax+by+c|}{\sqrt{a^2+b^2}}$ $\quad P\,(\overset{x}{-2}, \overset{y}{4})$

$= \dfrac{|x+2y+2|}{\sqrt{1^2+2^2}}$

$= \dfrac{|-2+2(4)+2|}{\sqrt{5}}$

$= \dfrac{|8|}{\sqrt{5}}$

$= \dfrac{8}{\sqrt{5}}$

v) $RS = \sqrt{85}$ $\quad P(\overset{x_1}{-2}, \overset{y_1}{4}) \quad Q(\overset{x_2}{4}, \overset{y_2}{1})$

Area $PQRS = \dfrac{h}{2}(a+b)$

$= \dfrac{8}{2\sqrt{5}}(\sqrt{85}+\sqrt{45})$

$= \dfrac{8\sqrt{85}}{2\sqrt{5}} + \dfrac{8\sqrt{45}}{2\sqrt{5}}$

$= 4\sqrt{17} + 4\sqrt{9}$

$= 4\sqrt{17} + 12$

b) $\displaystyle\int_1^3 f(x)\,dx = \frac{h}{3}\left[y_0 + y_n + 4(y_1+y_3) + 2(y_2)\right]$

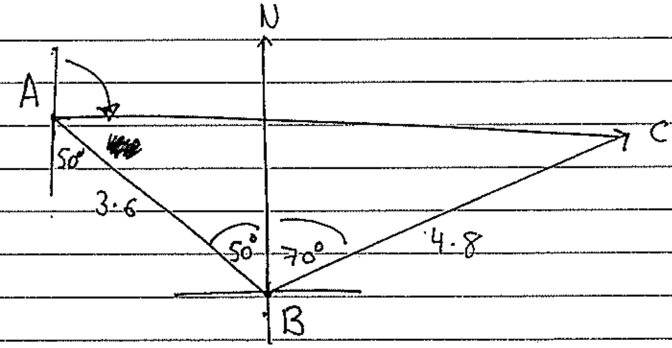
$= \dfrac{0.5}{3}\left[4+8 + 4(1.5+2.5) + 2(2)\right]$

$= \dfrac{1}{6}\left[12+16+4\right]$

$= \dfrac{32}{6}$

$= 5\frac{1}{3}$ units^2.

c)



i) After 2hrs

$AC^2 = 3.6^2 + 4.8^2 - 2(3.6\times4.8)\cos 120°$ Cosine Rule

$= 53.28$

$AC = 7.29931$

$= 7.3$ km (1dp) ②

ii) $\dfrac{\sin\angle BAC}{4.8} = \dfrac{\sin\angle ABC}{7.3}$ Sine Rule

$\sin\angle BAC = \dfrac{4.8\times\sin(120)}{7.3}$

$\angle BAC = 34.7°$

∴ Bearing is

$180 - 50 - 34.7$

$= 95.29°$

Question 13

a) $50 + 64 + 78 + \dots$ Arithmetic Series

i) $T_n = a + (n-1)d$

$= 50 + 14(n-1)$

$= 50 + 14n - 14$

$= 36 + 14n$

$T_{28} = 36 + 14(28)$

$= 428$

ii) $T_n > 800$

$36 + 14n > 800$

$n > 54.57$

∴ On the 55th Day

iii) $S_n = \frac{n}{2}(2a + (n-1)d)$

$10000 = \frac{n}{2}(100 + 14n - 14)$

$10000 = 43n + 7n^2$

∴ $7n^2 + 43n - 10000 = 0$

$n = \frac{-43 \pm \sqrt{43^2 + (4 \times 7 \times 10000)}}{2 \times 7}$

$n = \frac{-43 \pm \sqrt{530.89}}{14}$

$n = 34.84$ ∴ on the 35th Day

b) $2y = x^2 - 4x$

i) $2y = x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2$ Completing the Square

$2y + \left(\frac{-4}{2}\right)^2 = (x-2)^2$

$2y + 4 = (x-2)^2$

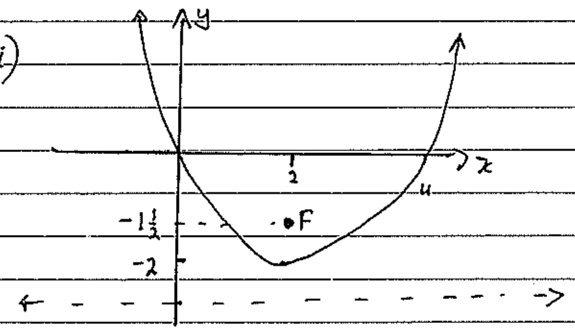
$2(y+2) = (x-2)^2$ (2)

∴ $(x-2)^2 = 2(y+2)$

∴ $4a = 2$ Vertex is $(2, -2)$

$a = \frac{1}{2}$

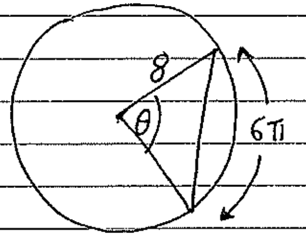
ii)



Focus $\left(2, -1\frac{1}{2}\right)$ (1)

iii) Directrix $y = -2.5$ (1)

c)



i) $l = r\theta$

$6\pi = 8\theta$

$\theta = \frac{3\pi}{4}$ (135°)

$\therefore \angle AOB = \frac{3\pi}{4}$ (1)

ii) $\text{Area} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2} \times 8^2\left(\frac{3\pi}{4} - \sin\left(\frac{3\pi}{4}\right)\right)$

$= \frac{1}{2} \times 64\left(\frac{3\pi}{4} - \frac{1}{\sqrt{2}}\right)$

$= 52.77$

$= 52.78 \text{ cm}^2$ (2 dp)

$= 52.8 \text{ cm}^2$ (1 dp) (2)

d) $\text{Area} = \int_0^2 (x^2+2)\,dx - \int_0^2 \sin x \cdot dx$

$= \int_0^2 (x^2+2-\sin x) \cdot dx$

$= \left[\frac{x^3}{3} + 2x + \cos x\right]_0^2$

$= \left[\frac{8}{3} + 4 + \cos(2)\right] - \left[\frac{0}{3} + 0 + \cos 0\right]$

$= 5.25 \text{ units}^2$ (3)

Question 14

a) $y = 2x^3 + 3x^2 - 12x + 9$

i) Stat. Pts when $y' = 0$

$y' = 6x^2 + 6x - 12$
$0 = 6x^2 + 6x - 12$
$0 = 6(x^2 + x - 2)$
$0 = 6(x+2)(x-1)$

$x = -2$ $x = 1$

• When $x = -2$ $y = 2(-2)^3 + 3(-2)^2 - 12(-2) - 9$
$= -16 + 12 + 24 - 9$
$= 11$

Pt at $(-2, 11)$ $y'' = 12x + 6$
$= 12(-2) + 6$
$= -18$ < 0 Concave Down
∴ Maximum

• When $x = 1$ $y = 2(1)^3 + 3(1)^2 - 12(1) - 9$
$= -16$

Pt at $(1, -16)$ $y'' = 12(1) + 6$
$= 18$ > 0 Concave Up
∴ Minimum

(ii) Point of Inflexion when $y'' = 0$

$y'' = 12x + 6$
$0 = 12x + 6$
$12x = -6$
$x = -\frac{1}{2}$

$y = 2(-\frac{1}{2})^3 + 3(-\frac{1}{2})^2 - 12(-\frac{1}{2}) - 9$
$= -2.5$

∴ Point of Inflexion Possible at $(-0.5, -2.5)$

Test:

x	-1	-0.5	0
y''	-6	0	6

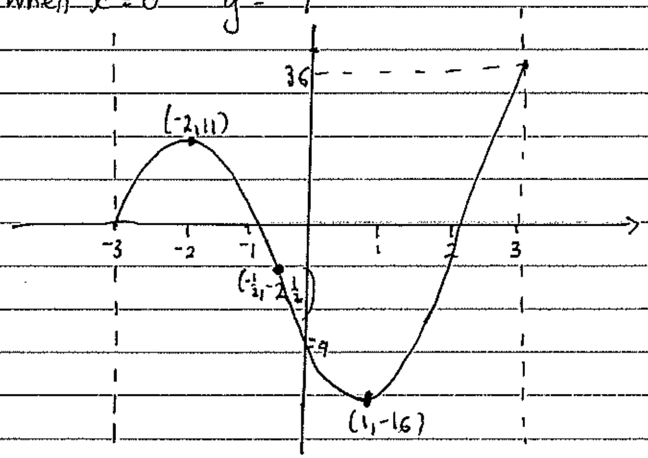
Change in Concavity
∴ It is a pt of inflexion

iii) Sketch $y = 2x^3 + 3x^2 - 12x - 9$

When $x = -3$ $y = 2(-3)^3 + 3(-3)^2 - 12(-3) - 9$
$= 0$

When $x = 3$ $y = 2(3)^3 + 3(3)^2 - 12(3) - 9$
$= 36$

When $x = 0$ $y = -9$



(iv) Minimum Value is -16

b) $P = 750\,000$

$r = 4.8\%$ p.a

$= 0.4\%$ per month

$= 0.004$ per month

$n = 30$ years

$= 360$ months

i) $A_1 = 750\,000(1.004) - M$

$A_2 = A_1(1.004) - M$

$= \left(750\,000(1.004) - M\right)(1.004) - M$

$= 750\,000(1.004)^2 - M(1.004) - M$

$= 750\,000(1.004)^2 - M(1 + 1.004)$

(ii) $A_3 = A_2(1.004) - M$

$= \left[750\,000(1.004)^2 - M(1 + 1.004)\right](1.004) - M$

$= 750\,000(1.004)^3 - M(1.004 + 1.004)^2 - M$

$= 750\,000(1.004)^3 - M\left(1 + 1.004 + 1.004^2\right)$

Continuing Pattern

$A_n = 750\,000(1.004)^n - M\underbrace{\left(1 + 1.004 + 1.004^2 + \cdots 1.004^{n-1}\right)}_{\text{Geometric Series}}$

$a = 1 \quad r = 1.004$

$S_n = \frac{1\left((1.004)^n - 1\right)}{\ldots}$

$\therefore \quad A_n = 750\,000(1.004)^n - M\left[\frac{1.004^n - 1}{0.004}\right]$ (2)

(iii) When loan repaid $A_n = 0$

$A_{360} = 750\,000(1.004)^{360} - M\left[\frac{1.004^{360} - 1}{0.004}\right] = 0$

$\therefore$

$750\,000(1.004)^{360} = M\left[\frac{1.004^{360} - 1}{0.004}\right]$

$M = \frac{750\,000(1.004)^{360}}{\left[\frac{1.004^{360} - 1}{0.004}\right]}$ (2)

$M = \$3934.99$

iv) $A_n = 750\,000(1.004)^n - 5000\left[\frac{1.004^n - 1}{0.004}\right]$

$0 = 750\,000(1.004)^n - 1250\,000(1.004^n - 1)$

$0 = 750\,000(1.004)^n - 1250\,000(1.004)^n + 1250\,000$

$0 = -500\,000(1.004)^n + 1250\,000$

$\therefore \quad 500\,000(1.004)^n = 1250\,000$

$1.004^n = \frac{1250\,000}{500\,000}$

$1.004^n = 2.5$

$\ln(1.004^n) = \ln(2.5)$

$n \ln(1.004) = \ln(2.5)$

$n = \frac{\ln(2.5)}{\ln(1.004)} \doteq 229.53$ months

Question 15

a) i) Area $= (w-2)(h-3.5)$ | Now $hw = 200$, $\therefore h = \frac{200}{w}$

$= (w-2)\left(\frac{200}{w} - 3.5\right)$

$= 200 - 3.5w - \frac{400}{w} + 7$

$= 207 - 3.5w - \frac{400}{w}$ (2)

ii) $A = 207 - 3.5w - 400w^{-1}$

$A' = -3.5 + 400w^{-2}$

$= -3.5 + \frac{400}{w^2}$ ✓

Stat. Pts when $y' = 0$

$3.5 = \frac{400}{w^2}$

$w^2 = \frac{400}{3.5}$

$w = \pm\sqrt{\frac{400}{3.5}}$

$w = \pm 10.69$ cm.

Neg. Value doesn't make sense

So $w = 10.69$cm ✓

• Test nature

$y'' = -800w^{-3}$

$= -800(10.69)^{-3} < 0$ Concave Down $\therefore$ Maximum

• If $w = 10.69$

$h = \frac{200}{10.69}$

$h = 18.71$ cm. (3)

b) $Q = Q_0 e^{-kt}$

i) When $t = 0$ $5 = Q_0 e^{-0}$

$\therefore Q_0 = 5$

$\therefore$

$Q = 5e^{-kt}$

When $t = 12$, $Q = 2.5$

$2.5 = 5e^{-k \times 12}$

$\frac{1}{2} = e^{-12k}$

$\ln\left(\frac{1}{2}\right) = -12k$

$k = $ ~~0.0578524830~~

$\doteqdot 0.05776226505$

(i) Find t when $Q = 0.5$

$$0.5 = 5e^{-kt}$$

$$\frac{0.5}{5} = e^{-kt}$$

$$\ln\left(\frac{0.5}{5}\right) = -kt$$

$$t = \frac{\ln\left(\frac{0.5}{5}\right)}{-k} = \frac{\ln\left(\frac{0.5}{5}\right)}{\frac{\ln(0.5)}{12}}$$

$t = 39.863$ hours

$t = 40$ hours (2)

c) $x = t^3 - 12t^2 + 36t - 8$

(i) At rest when $\dot{x} = 0$

$$\dot{x} = 3t^2 - 24t + 36$$

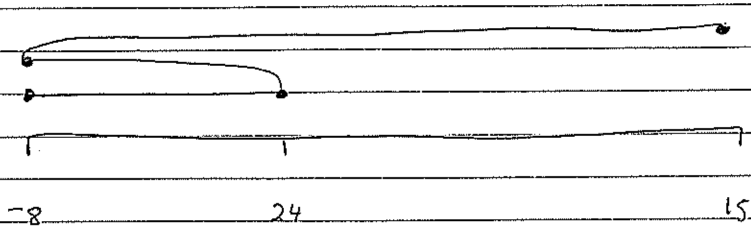
$$0 = 3(t^2 - 8t + 12)$$

$$0 = 3(t-6)(t-2)$$

$t = 6$ $t = 2$ (2)

(ii) First changes direction at $t = 2$ (1)

(iii) Find total distance travelled in first 10 s



at $t = 0$ $x = -8$
$t = 2$ $x = 24$
$t = 6$ $x = -8$
$t = 10$ $x = 152$

Total dist = $32 + 32 + 160$
$= 224$ m. (2)

16

a)

$$LHS = \frac{\sin^2 x}{1-\cos x} + \frac{\sin^2 x}{1+\cos x}$$

$$= \frac{\sin^2 x(1+\cos x) + \sin^2 x(1-\cos x)}{(1-\cos x)(1+\cos x)}$$

$$= \frac{\sin^2 x(1+\cos x + 1 - \cos x)}{1-\cos^2 x}$$

$$= \frac{\sin^2 x\,(2)}{\sin^2 x}$$

$= 2$

$= RHS.$

b) $\frac{dV}{dt} = \frac{1}{100}(30t - t^2)$

i) at $= 15$

$$\frac{dV}{dt} = \frac{1}{100}(30(15) - 15^2)$$

$$= \frac{1}{100}(450 - 225)$$

$$= \frac{225}{100}$$

$= 2.25$ cm^3/min

ii) $\int \frac{dV}{dt} = \frac{1}{100}\int(30t - t^2)\,dt$

$$V = \frac{1}{100}\left(15t^2 - \frac{t^3}{3}\right) + c$$

at $t=0, V=0$ $\therefore c = 0$

$$V = \frac{1}{300}(45t^2 - t^3)$$

at $t=15$,

$$V = \frac{1}{300}(45(15)^2 - 15^3)$$

$= 22.5$ cm^3

c) $\ddot{x} = 4\sin 2t$ at $t=0, x=4, \dot{x}=0$

i $\dot{x} = \int \ddot{x}\,dt$

$$= \int 4\sin 2t\,dt$$

$$= \frac{-4\cos 2t}{2} + c.$$

at $t=0, \dot{x}=0$

$0 = -2\cos 2(0) + c$

$= -2 + c$

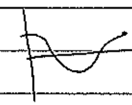
$c = 2$

$\therefore \dot{x} = -2\cos 2t + 2$

ii for rest $\dot{x}=0$

$-2\cos 2t = -2$

$\cos 2t = 1$



$2t = 0, 2\pi, 4\pi, \ldots$

$t = 0, \pi$

c) iii) $x = \int \dot{x}\, dt$

$= \int (-2\cos 2t + 2)\, dt$

$= \frac{-2\sin 2t}{2} + 2t + c$

at $t=0$, $x=4$

$4 = -\sin 2(0) + 2(0) + c$

$= c$

$\therefore \quad x = -\sin 2t + 2t + 4.$

d) $f(x) = \frac{1}{2}(e^{x} + e^{-x})$

$f(-x) = \frac{1}{2}(e^{-x} + e^{-(-x)})$

$= \frac{1}{2}(e^{x} + e^{-x})$

$= f(x) \quad \therefore$ even function

$V = \pi \int_{-2}^{2} \left[\frac{1}{2}(e^{x} + e^{-x})\right]^2 dx.$

$= \frac{\pi}{4} \times 2 \int_{0}^{2} (e^{x} + e^{-x})^2 dx$

$= \frac{\pi}{2} \int_{0}^{2} (e^{2x} + 2e^{x}.e^{-x} + e^{-2x})\, dx$

$= \frac{\pi}{2} \left[\frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2}\right]_{0}^{2}$

$= \frac{\pi}{2} \left(\frac{e^{2(2)}}{2} + 2(2) - \frac{e^{-2(2)}}{2} - \left(\frac{e^{2(0)}}{2} + 2(0) - \frac{e^{-2(0)}}{2}\right)\right)$

$= \frac{\pi}{4}\left((e^{4} - e^{-4} + 8) - \left(\frac{1}{2} - \frac{1}{2}\right)\right)$

$= \frac{\pi}{4}(e^{4} - e^{-4} + 8) = 49.15$

Multiple Choice Summary

1) C
2) A
3) C
4) C
5) D
6) A
7) C
8) B
9) C
10) C